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Applications of Quantum Principles

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Abstract

The departure from classical physics in the early 20th century was heralded by the discovery of a number of seemingly counter intuitive experimental principles. Movement at the atomic level was shown to be inherently random, given only by probabilities; energy at specific frequencies was quantised to discrete and non-divisible values; light and electromagnetic radiation were demonstrated to have properties of both waves *and* particles under different circumstances and at different times. These ideas and their implications are quite disconnected from our everyday view of the world, and their practical and technological value is often indistinct; quantum mechanics is often viewed merely as a mathematical means of describing the universe.

In the last century, however, it has become far more clear how a quantum perspective can be applied to both technology and practical physics – from accurate detection methods and photo-multiplication, to theoretical computers so powerful as to entirely deprecate Moore's Law[1]. I aim to explore some of the fundamental principles of quantum mechanics, and show not only how they came into popular consensus, but also how they can be applied in numerous technologies and scientific methods, both existing, historical and hypothetical.

Planck, Black Bodies and the Quantisation of Energy

One scientific predicament at the beginning of the 20th century was that of radiation from black-bodies. A black-body is a theoretical object which absorbs all light that falls upon it: it is therefore an ideal emitter of thermal radiation, although a perfect black-body has never been observed. The specific problem was how the intensity of radiation released from black-bodies relates to both its frequency and the temperature of the body.

Lord John Rayleigh and James Jeans proposed that the electromagnetic spectrum making up a black body is a series of standing waves: oscillators which could absorb or release energy at any frequency. They assumed that there was no limiting factor on the range of possible modes of oscillation – thus not only could any frequency of light could be released, but each frequency was equally likely.

Classical theory stated that the number of possible modes in a three-dimensional cavity is proportional to the square of the waves' frequency, and equipartition theory stated that the average energy of said oscillators should be equivalent to $kT/2$ [2]. Basing their theory on these principles, Rayleigh and Jeans proposed the distribution of spectral radiance (energy per unit volume per unit frequency – or more simply, energy density) from black-body electromagnetic radiation to be proportional to the square of the frequency, for any temperature. They conjectured that the radiance distribution followed the following law, where s_ν is a function of frequency:

$$s_\nu = \frac{8\pi\nu^2}{c^3}kT$$

However when compared with experimental values, their law only appeared to work for low frequencies – and it is simple to see why. Their distribution diverges towards infinity – meaning that the total energy of a system tends towards infinite amounts, in what was dubbed the "Ultraviolet Catastrophe". If we consider temperature to be constant, a loose graphical interpretation of this law can be shown simply with the relation:

$$s_\nu \propto \nu^2$$

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[1] Moore, Gordon (1965) *Cramming More Components Onto Integrated Circuits*

[2] Baierlein, Ralph (1999) *Thermal Physics*

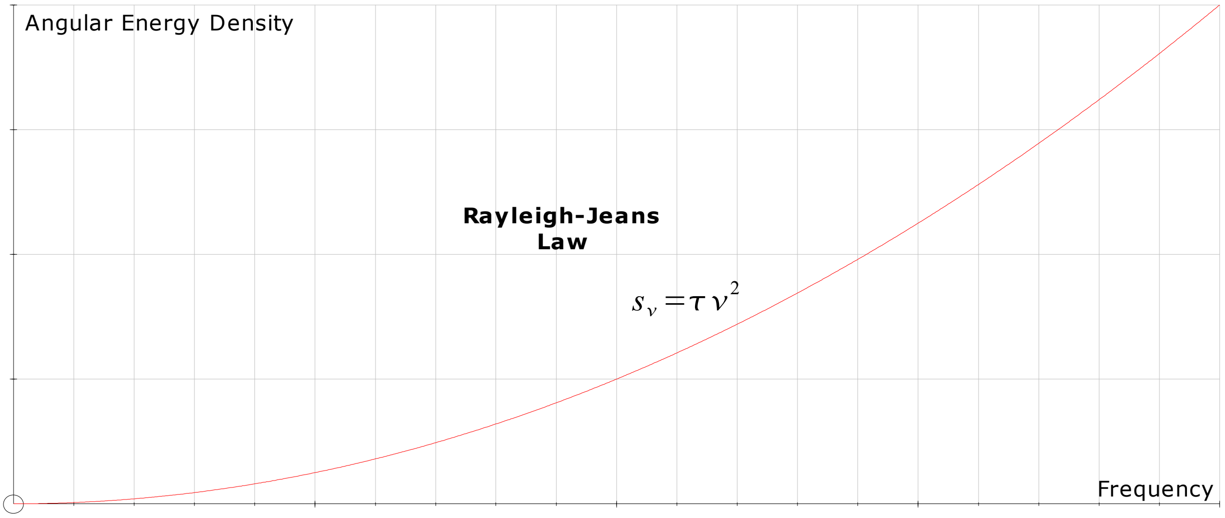

Fig 1 Rayleigh-Jeans Law – Black-Body Radiation Spectral Radiance and Frequency (τ is an arbitrary constant)

It is apparent that integrating this function over all frequencies would yield an infinite energy – which is obviously impossible. Despite its basis on classical physics, Rayleigh and Jeans' theory only worked at low frequencies – and aside from the so-called ultraviolet-catastrophe, their law disagreed with all experimental evidence[3] when $h\nu/kT > 1$

Max Planck took a different approach to the problem. Based on the assumption that the radiation could not be released continuously and only in packets governed by the threshold kT, or 'quanta', he theorised that the number of modes would decrease at higher frequencies, tending towards zero as the wavelength became smaller. This solved the classical 'infinite energy' problem, and predicted that the amount of energy in a black-body was indeed finite. Planck proposed the following electromagnetic formula to describe the spectral radiance:

$$s_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT} - 1}$$

Again, if we take temperature as a constant, Planck's Law can be simply demonstrated graphically with the relation:

$$u_\nu \propto \frac{\nu^3}{e^\nu - 1}$$

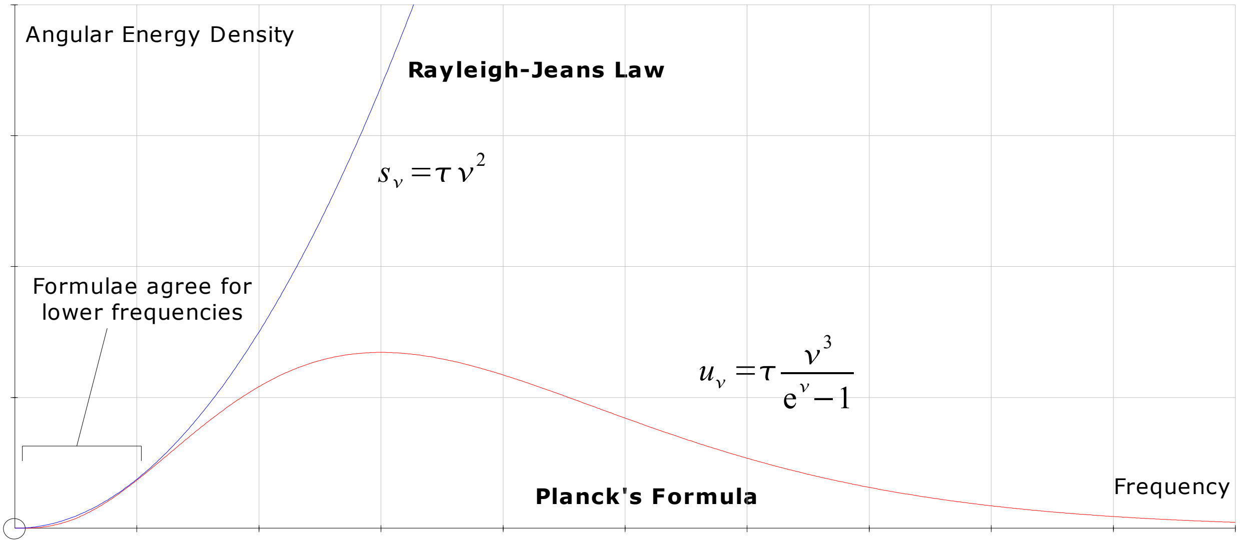

Fig 2 Planck's Radiation Formula and Rayleigh-Jeans Law (τ is an arbitrary constant)

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[3] Kragh, Helge (2000) *Max Planck: The Reluctant Revolutionary*

It is possible to see graphically how the two laws agree for lower frequencies – and also that the integral of Planck's function would yield a finite overall energy. In deriving this law, however, Planck implied one catch: the relation $E=h\nu$ suggests that at a constant frequency, the radiation's energy could only ever be a multiple of *h.* The Planck constant, *h* = 6.606 x 10^{-34} *js,* represents the amount of energy carried by a photon at 1Hz*.* In other words, energy can be quantized, and take only discrete rather than continuous values. This was one of the first steps towards a quantum interpretation of physics: Planck's 'packets' of light later became known as photons, and his ideas have prevailed despite being described by Planck himself as "a purely formal assumption" and even "an act of desperation"[4] intended purely as an empirical means to describe a system without infinite energy as a result.

Systems like black-bodies have been described as a "photon gas", due to their distribution of energy much like that seen in non-relativistic matter – Planck's distribution is hugely similar to the Maxwell-Boltzmann distribution[5], which depicts the velocities (and correspondingly, the energies) of particles in a mixture of moving particles at a given temperature.

Lasers

Since the idea was popularised by Einstein, considering light as particles or photons (with energies relating to Planck's $E=h\nu$) has yielded some important technological advances – one example being lasers. Lasers amplify light based on a process attributed to photons, known as "stimulated emission". An electron in a high energy state can be stimulated into jumping to a lower energy level by a passing photon: this transition causes an identical photon to be released, for the purpose of energy conservation. Because the two photons have the same frequency, phase, direction and polarisation, they are almost completely coherent – in other words, they can positively interfere with each-other, ultimately resulting in light with a higher intensity.[6] This is, importantly, very different to spontaneous emission, which also results in emitted photons, but in random directions; thus any coherence is unlikely. Although stimulated emission results in coherent photons, there is an extremely small distribution of frequencies emitted due to the Doppler effect. This occurs because the particles have temperature, equivalent to kinetic energy – their random movement causes the small overall variance in wavelengths for the photons released.

Lasers work by focusing photons through a material which has a large proportion of electrons residing in higher levels – this material is known as the "active laser medium", and can consist of certain types of crystals, ionic transition metals or gases. The electrons in this material are stimulated with an external energy source prior to being bombarded with photons; this caused a "population inversion" in which more atoms exist in an excited state, meaning there is a higher probability that they will emit photons rather than absorb them[7]. After this, photons are focused and reflected a number of times through the material, before they are released at a greater intensity. Naturally, for the resulting light to be coherent, the incident light must also be in phase.

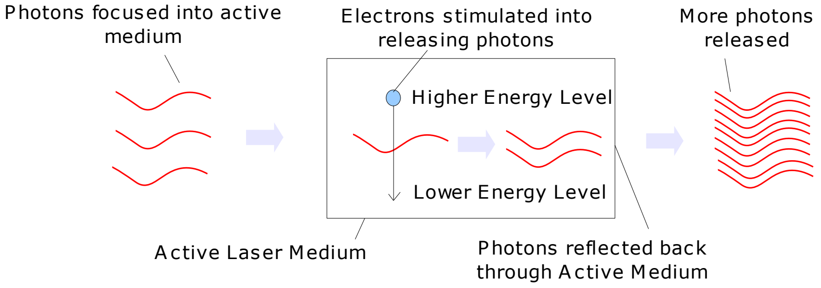

Fig 3 Laser - Stimulated Emission in Active Medium

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[4] Kantorovich, Aharon (1993) Scientific Discovery: Logic and Tinkering
[5] Roos, Matts (2003) *Introduction to Cosmology*
[6] Balkanski, Minko and Wallis, Richard (2000) *Semiconductor Physics and Applications*
[7] Csele, Mark (2004) *Fundamentals of Light Sources and Lasers*

The Photoelectric Effect

Using a wave based model, it might be assumed that given a constant stream of electromagnetic radiation, electrons in a metallic plate would eventually gain enough energy to break free from the surface. The amount of energy gained by these electrons should depend largely on the intensity of the radiation; the more radiation absorbed, the more energy the particles would gain.

This explanation does not hold true for experiments: in reality, the intensity only influences the number of electrons emitted – and it is the frequency of the light absorbed that is proportional to the particles' energy, as Planck's formula $E=h\nu$ predicts. This fact can be simply demonstrated, even with visible light – using potassium as an example.

Potassium has a work function of 2.3eV – meaning that in theory, a photon with a minimum energy 2.3eV must transfer its energy to a free electron in this metal, in order for it to have the necessary energy to be released.

If we take the opposite ends of the visible electromagnetic spectrum, it is simple to find the energy of an individual photon of violet and red light using the relation $E=h\nu$

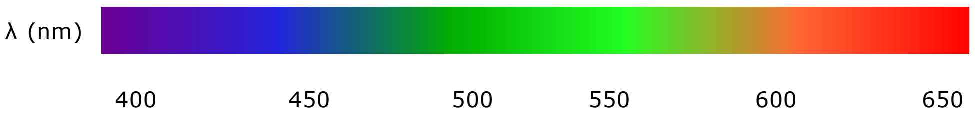

Fig 4 Visible Light Spectrum

$$Energy(eV)=\frac{h\cdot\nu}{e}=\frac{h\cdot c}{e\cdot\lambda}$$

Violet Light:

$$E=\frac{[6.6\times10^{-34}]\cdot[3\times10^{8}]}{[1.6\times10^{-19}]\cdot[400\times10^{-9}]}=3.1\text{eV}$$

Red Light:

$$E=\frac{[6.6\times10^{-34}]\cdot[3\times10^{8}]}{[1.6\times10^{-19}]\cdot[650\times10^{-9}]}=1.9\text{eV}$$

This demonstrates that violet light – but not red light – carries sufficient energy to free electrons from Potassium. In fact the actual necessary wavelength can easily be determined, given the work function of 2.3eV:

$$\lambda=\frac{h\cdot c}{E\cdot e}=\frac{[6.6\times10^{-34}]\cdot[3\times10^{8}]}{[2.3]\cdot[1.6\times10^{-19}]}=539\times10^{-9}m=539\,nm$$

This shows that green light with a maximum wavelength of 539nm would be required to free electrons from the surface of Potassium. This is characteristic of light behaving as a particle: in actuality, an electron absorbs the energy from a single photon; if this absorbed energy is greater than the work function of the material (defined as the amount of energy necessary to move an electron to immediately outside the solid's surface), the electron is emitted.

One example of the use of this effect in technology is in photomultipliers, within telescopes and night vision devices. These are necessary for dark environments – or when the intensity of a light source needs to be increased for the benefit of the human eye. The low intensity light is focused onto a very thin layer of gallium arsenide, which has a very high photosensitivity in the spectral region of about 450-950nm[8]. Due to the photoelectric effect, the energy of emitted electrons depends only upon the frequency of the light quanta. The emitted electrons are accelerated to a sufficiently high energy using an electric field, and are then used to cause another cascade of a greater number of electrons from a secondary photoconductor. These electrons are then accelerated enough to excite a phosphor screen, causing photons to be released, which are then focused into the eyepiece of the user, or a detection system.[9][10]

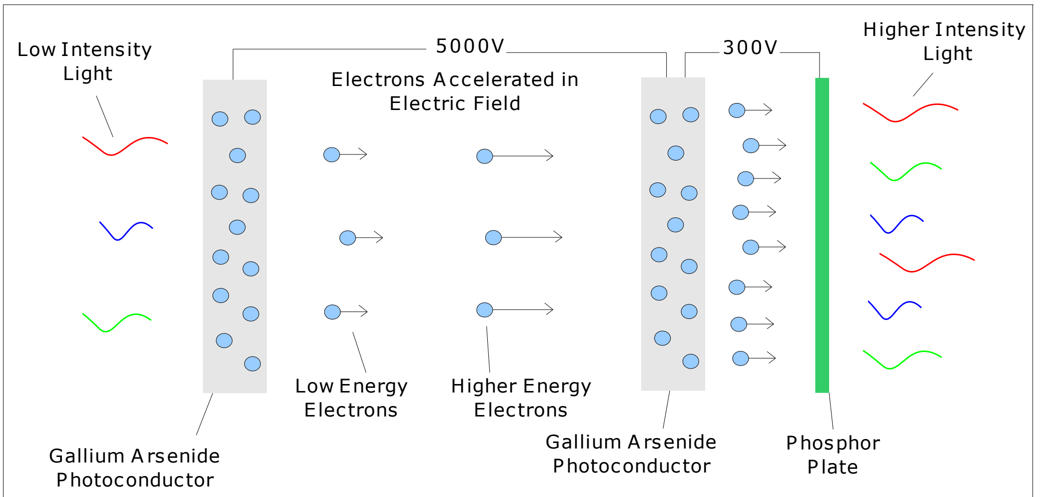

Fig 5 Photomultiplier

Quantum Superposition and Single-Particle Interference

Seeing light from a quantum perspective yields a number of other principles which do not adhere quite as closely to standard logic as the photoelectric effect, as an example: its implications on superposition and interference, for instance, are almost entirely counter-intuitive.

Consider a stream of photons, split by a half-silvered mirror and then detected in their respective directions:

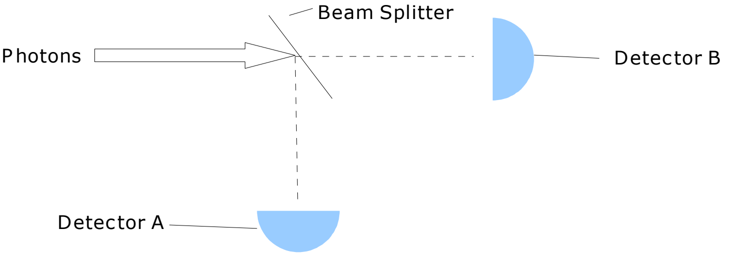

Fig 6 Single-Particle Interference with Single Beam-Splitter

As would be expected, the proportion of photons detected by each detector is statistically equal. Classical mechanics explains this phenomenon with the logical reason – that half of the light is directed to either detector. However, quantum mechanics states that instead of simply taking one path, each photon takes both – and it is only when the photon is detected that the number of paths it has taken collapses to one[11].

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[8] Westminster International Limited (2002) *How Night Vision Works*
[9] Burble Industries inc. (1989) *Photomultiplier Handbook*
[10] Wikimedia (2006) *Night Vision Device - Photomultiplier*
[11] West, Jacob (2000) *An Introduction to Quantum Computing*

This effect is demonstrated more clearly by adding another beam-splitter into the arrangement.

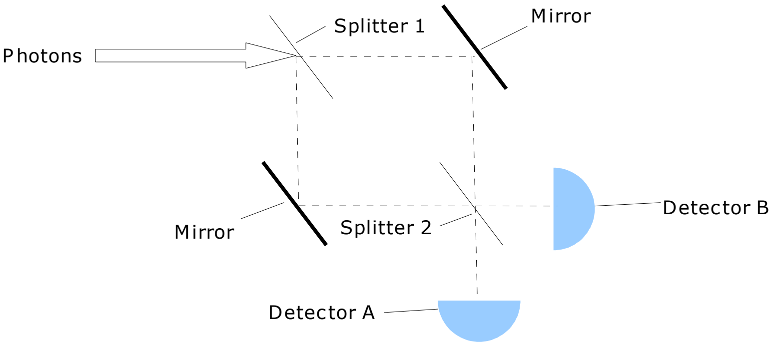

Fig 7 Single-Particle Interference with Double Beam-Splitter

Again using classical ideas about light, it could be assumed from this arrangement that each detector would register half of the light entering the system. Where $P(S_n)=0.5$ is the probability that the photons go in a specific direction upon leaving a given beam splitter, the probability of light entering either detector should simply be $2P(S_1 \cap S_2)=2(0.5^2)=2(0.25)=0.5$. But this rule is not observed in experiments – the photons are all detected at Detector A, and none appear to arrive at B. The explanation for this can only be that both paths were taken simultaneously, and quantum interference, or superposition, occurred at the point of intersection (at the second beam-splitter) making the probability of the photon reaching Detector B impossible. This is further confirmed when either path is blocked by, for example, an absorbing screen – in this case, both detectors will once more have an equal chance of registering a photon. Alongside entanglement, this is one premise which is used in developing quantum computing – which will be discussed later.

Heisenberg's Uncertainty Principle

In 1927, Werner Heisenberg proposed that simultaneously knowing the exact position and momentum of a particle is inherently impossible – this is neither due to inaccurate measurements nor the observer effect (where the observer influences a system by observing it) – and is more a property of nature than an artifact of experimentation[12]. The principle can be relatively simply described using probability.

Heisenberg's principle states that the multiples of the root mean square deviation (a value which represents the typical difference between an expected or mean value, and the actual values observed) of the particle's position and momentum will always be greater than half of Dirac's constant[13] (where Dirac's constant is $h/2\pi$ and represents joules per radian per second, rather than joules per hertz). So:

$$\sqrt{\frac{\sum x^2 - n\bar{x}^2}{n}} \times \sqrt{\frac{\sum p^2 - n\bar{p}^2}{n}} \geq \frac{\hbar}{2}$$

Or far more simply:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

Importantly, this suggests that as one RMSD becomes smaller, the other must increase in size as the product approaches $\hbar/2$ - meaning that absolute accuracy for either value is impossible[14]. This becomes relevant in the real world when, as an example, we observe electrons in atomic orbitals (discussed in 'Wave Functions'). Classical physics

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[12] Chauhan, Bhag (2008) *Science in Trauma*
[13] McWeeny, Roy (2003) *Quantum Mechanics: Principles and Formalism*
[14] Bohm, David (1989) *Quantum Theory*

has been described as the limit of quantum mechanics, when *h* tends to zero: this is further demonstrated by Heisenberg's principle; when *h* becomes negligible, the amount of uncertainty for either variable is minimal – i.e. position and momentum can take on relatively low root-mean-deviations, for larger systems. The implications of the uncertainty principle in the real world are largely philosophical: it would seem based on Heisenberg's rule that on the atomic scale, no future event can be predicted to an infallible degree of accuracy, therefore causality on the fundamental level is relatively inaccessible. Importantly, this results in an indeterminate universe – which may prove to be a restriction on technology, as it approaches the quantum level in computing and detection.

Wave Functions and Probability Amplitudes

Quantum systems are normally described mathematically as a series of probabilities, mapped by a wave-function. This function describes a system or a particle's position, momentum, spin, etc, as a complex probability distribution, where the variables are all complex numbers involving i (where $i^2=-1$).

In the 1930s, Paul Dirac proposed a system for representing quantum probabilities, known as 'Dirac' or 'bra-ket' notation. This is split into what is known about the current state of a system – normally the initial stage in an event or a transition- and a final state of said system. The ket, for example, might be expressed as follows:

$|v\rangle$ A particle with velocity v
$|v=2.7\rangle$ A particle with velocity equal to 2.7

Typically a ket will contain a large number of variables – thus it is more commonly written as a state vector $|\Psi\rangle$ which represents a system in the state Ψ, and consists of all the components of the system as a column vector.

The bra represents a possible final state of the depicted system – and when put together, the bra and the ket give a probability amplitude for the system in the desired state.

For example, $\langle p=4.5|\Psi\rangle$ represents the probability amplitude that a particle in state Ψ will have momentum 4.5 and the square of this amplitude – in this case $|\langle p=4.5|\Psi\rangle|^2$ - gives the probability density for its occurrence[15].

Wave-functions map out all of the possible results and probabilities in a quantum system, but when the mapped event happens, what is known as 'wave-function collapse' occurs. This is best described as a "transition from the potential to the actual"[16] - although whether it is a fundamental quantum phenomenon or not is debatable. Either way, it is the change from a state of mere probability to a state of definite knowledge.

It has been argued that wave-function collapse, specifically the idea of a system tuned by observance, is only a subjective phenomenon. In 1957, in his 'relative-state' formulation, Hugh Everett proposed that quantum decoherence is a mechanism by which new universes to come into existence - after a quantum interaction has been determined. Everett argued that far from being just a mathematical description of a particle, the wave-function actually is the particle – and vice-versa[17]. This appears to solve a number of the philosophical problems with the theory – one such being Schroedinger's Cat, which until it is observed is left in a superposition between life and death after the potential decay of a single nucleus -- according to Everett, the cat is both alive and dead, in different universes. This interpretation also reconciles the indeterminacy of quantum events, resolving Einstein's famous scruple with the theory wherein he claimed that "God does not play dice with the universe". The issues of determined causality that the relative-state interpretation suggests, however, are equally as profound as the alternative.

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[15] Rioux, Frank (2007) *Elements of Dirac Bracket Notation*
[16] Kiefer, Claus (2002) *On the interpretation of quantum theory – from Copenhagen to the present day*
[17] Everett, Hugh (1957) *Relative State Formulation of Quantum Mechanics*

One example of the relevance of wave-functions is electrons in an atomic orbital. The atomic orbital itself is simply the wave-function of the electron – its shape is essentially a probability cloud or distribution for the varying positions and quantum states the electron is likely to take relative to the nucleus. An electron can only take certain energies, each of which corresponds to different orbital levels – however an exact knowledge of an electrons' energy, according to Heisenberg's Uncertainty Principle, means that calculation of its exact position is impossible – hence the probability 'shell'.[18]

Quantum Computing

Traditionally, computers store and manipulate information by representing data in binary form - as a series of 1s and 0s – and then performing boolean operations on it via a series of logic gates – for example AND, OR, XOR, NOT ... each coherent operation carried out on a computer, from simple addition to text-parsing, is simply a number of levels of abstraction away from binary logic.

A binary value, simply being a 1 or a 0, is known as a 'bit' – similarly, a quantum-bit is known as a 'qubit'. A qubit can take the values 1, 0, or exist as a superposition between both – although when a qubit's state is measured, it must take one of the two previous definite values. Rather than being passed through Boolean logic gates, qubits are manipulated by quantum gates which perform a unitary quantum transformation on the particle, the final measurement being the computational result.

The added computational power from quantum superpositions can be demonstrated as follows: taking 10 ordinary bits, the number of possible configurations of said data – given the possible number of states for each bit as being 2 – can be expressed as: 2^{10} - 0000000001, 0000000010, etc. In a set of 10 qubits, there can be a total of 2^{10} possible states of superposition – each of which has 2^{10} possible configurations of 1s and 0s. Aside from this, any operation on a given qubit would simultaneously affect each of the 2^{10} states – meaning that far less time needs to be taken for computation at an exponential rate relative to the number of operations per second.

Only considering the number of possible states of bits and qubits, the power of quantum computers can be simply compared to standard binary computers with a simple pair of functions, where P_n is a function of computing power relative to the number of operations (boolean and unitary) per second for a given number of bits or qubits:

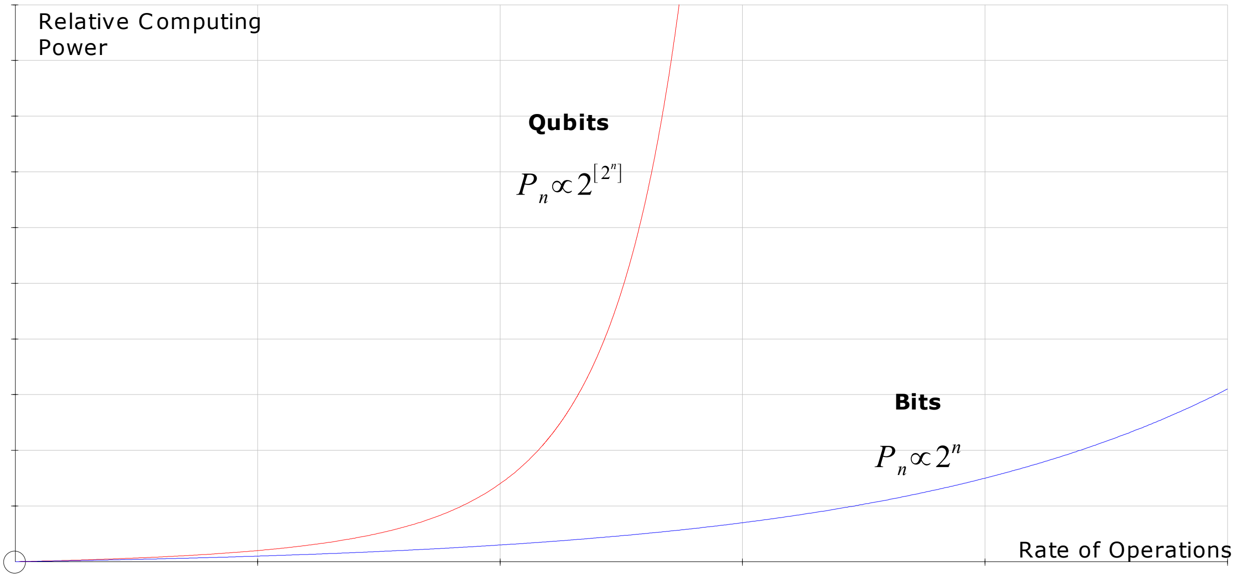

Fig 8 Computing power relative to traditional binary bits and quantum bits (qubits) per operation

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[18] Kotz, Treichel, Weaver (2005) *Chemistry and Chemical Reactivity*

One obstacle for quantum computing is the principle of decoherence: that is, any small interaction with the ambient environment might cause a qubit to choose a definite state, destroying any superposition. The number of particles making up a qubit is inversely proportional to its decoherence time – and at present, the decoherence rate is at roughly five-hundred nanoseconds. It is estimated that to be truly useful in computing, any time approaching one second would be useful, during which a great number of quantum operations are possible, and after which other factors would limit the qubits usefulness anyway[19]. Decoherence would also be a major obstacle to storing data in a quantum manner; the lifespan of which would be limited to seconds as opposed to years.

Decoherence is essentially equivalent to wave-function collapse in that the particle superposition takes on a fixed value – however this information serves no purpose for the observer. In fact, it is only the illusion of wave-function collapse that is demonstrated with decoherence – the total superposition still exists, but is beyond any possible measurement.

Entanglement

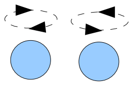
According to quantum mechanics, when two particles interact, they become irrevocably linked – so much so that it makes little sense to talk about one without reference to its counterpart. This phenomenon results in what seems to be superluminal communication – although this can not be manipulated so far as to affect causality. Entanglement plays a large role in quantum information theory, not because of its appearance to violate general relativity, but because it can be used to encode more information to a single particle than traditional bits. A qubit could normally only carry one bit of information at a given time – however, if it assumed to be entangled with another particle, more information can be derived from it without need to transmit its counterpart – thus its efficiency is doubled. This technique is normally referred to as "superdense coding", and is illustrated loosely as follows:[20]

1. Two quantum particles are entangled:

2. The particles are then separated, one given to the source of the information and one to the recipient:

3. When the information needs to be transmitted, a quantum operation is performed on the first particle, which causes a corresponding change in the second:

4. The first particle is then transmitted to the second (abiding to conventional relativistic laws), where both are measured:



5. This results in two 'bits' of data, encoded and transmitted using one particle: hence the 'superdense' coding.

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[19] Collins, Graham (2005) *Quantum Bug*

[20] Hirvensalo, Mika (2004) *Quantum Computing*

Conclusion

Sources

1. Gordon Moore - *Cramming More Components Onto Integrated Circuits* – Published 1965, Electronics Magazine
2. Ralph Baierlein – *Thermal Physics* – Published 1999, Cambridge Press
3. Helge Kragh – *Max Planck: The Reluctant Revolutionary* – Published 2000, physicsworld.com
4. Aharon Kantorovich - *Scientific Discovery: Logic and Tinkering* – Published 1993, Suny Press
5. Matts Roos - *Introduction to Cosmology* – Published 2003, Wiley
6. Minko Balkanski and Richard Wallis - *Semiconductor Physics and Applications* – Published 2000, Oxford Press
7. Mark Csele - *Fundamentals of Light Sources and Lasers* – Published 2004, Wiley
8. *How Night Vision Works* – Published 2002, Westminster International Limited
9. *Photomultiplier Handbook* – Published 1989, Burble Industries inc.
10. *Night Vision Device: Photomultiplier* - Published 2006, Wikimedia Foundation
11. Jacob West - *An Introduction to Quantum Computing* - Published 2000, cs.caltech.edu
12. Bhag Chauhan - *Science in Trauma* - Published 2008, Lisbon University
13. Roy McWeeney - *Quantum Mechanics: Principles and Formalism* - Published 2003, Courier Dover
14. David Bohm - *Quantum Theory* - Published 1989, Courier Dover
15. Frank Rioux - *Elements of Dirac Bracket Notation* - Published 2007
16. Claus Kiefer - *On the interpretation of quantum theory – from Copenhagen to the present day* - Published 2002, University of Cologne
17. Hugh Everett - *Relative State Formulation of Quantum Mechanics* - Published 1957, Princeton University
18. Kotz, Triechel, Weaver - *Chemistry and Chemical Reactivity* - Published 2005, Thompson Brooks
19. Graham Collins - *Quantum Bug* - Published 2005, Scientific American
20. Mika Hirvensalo - *Quantum Computing* - Published 2004, Springer

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